Model equations for planetary and synoptic scale atmospheric motions associated with different background stratification

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Abstract :

We consider planetary scale atmospheric motions (spatial scales comparable with the earth's radius and a corresponding advective time scale of the order of ten days) and the synoptic eddies (characteristic length and time scales: $1,000\,\mathrm{km}$ *and* $1\,\mathrm{day}$*, respectively). We present reduced equations that aim to capture the relevant dynamics on the planetary scale and account for multiscale planetary-synoptic interactions. The derivation of the equations is based on an unified multiple-scales asymptotic approach. We examine two different flow regimes associated with different background stratification.*

Key-words :

reduced atmospheric model ; planetary-synoptic interactions

1 Introduction

Observations show the existence of large number of low-frequency atmospheric regimes (periods longer than 10 days) with planetary spatial scales (scales larger than 3000 km), which have an important contribution to the variability of the atmosphere (e.g. the thermally and orographically induced quasi-stationary planetary Rossby waves; teleconnection patterns; mean meridional circulations (Hadley, Ferrel and the polar cells); zonally mean flows (subtropical and polar jets)). On the other hand the local wind fields and precipitation patterns are influenced by synoptic disturbances (1 day time scale and spatial scales of 1000 km). Both types of regimes - on the planetary and on the synoptic scales, are essential for the weather variations and also for the climate, since they are responsible for the heat, momentum and water vapor transport in the atmosphere. We use an unified multiple scales asymptotic approach in order to derive reduced model equations describing the relevant atmospheric phenomena on the planetary and synoptic scales. The method is presented in Klein (2004), for some applications see Majda & Klein (2003); Klein & Majda (2006). This technique allows one in a systematic way to capture the important interactions between the two scales. Considering processes on the planetary scales we have to take into account effects due to the sphericity of the earth, large variations of the Coriolis parameter. We examine two regimes, associated with different background stratification.

2 The Planetary Regime

In the first regime, we assume horizontal velocities of the order of $10\,\mathrm{m/s}$ and relatively weak background potential temperature variations comparable in magnitude to those adopted in classical quasi-geostrophic theory. In the case, when we rescale the coordinates for planetary scales motions only, the asymptotic expansion of the potential temperature and of the horizontal wind reads

$$\Theta(\underbrace{\varepsilon^3 t}_{t_P}, \lambda_P, \phi_P, z) = 1 + \varepsilon^2 \Theta^{(2)} + \mathcal{O}\big(\varepsilon^3\big) \ ,$$
$$\boldsymbol{u}(t_P, \lambda_P, \phi_P, z) = \boldsymbol{u}^{(0)} + \varepsilon \boldsymbol{u}^{(1)} + \varepsilon^2 \boldsymbol{u}^{(2)} + \mathcal{O}\big(\varepsilon^3\big) \ ,$$

where λ, ϕ, z and t denote longitude, latitude,altitude and time, the subscript P marks variables resolving the planetary scales and ε is a small parameter, introduced in Klein (2004) (for details see also Majda & Klein (2003); Klein & Majda (2006)). All other dependant variables are expanded in a similar manner and substituted in the governing equations. We obtain as a leading order system the classical planetary geostrophic equations (PGEs): for the ocean see Robison & Stommel (1959); Welander (1959) and for the atmosphere see Phillips (1963); Burger (1958). They consist of geostrophic and hydrostatic balance, divergence constraint and a transport equation for the potential temperature

$$\frac{\partial}{\partial z}\boldsymbol{u}^{(0)} = \frac{1}{f}\boldsymbol{e}_z \times \nabla_P \Theta^{(2)} \, , \tag{1}$$
$$\nabla_P \cdot \rho^{(0)} \boldsymbol{u}^{(0)} + \frac{\partial}{\partial z}\rho^{(0)} w^{(3)} = 0 \, , \tag{2}$$
$$\frac{\partial}{\partial t_P}\Theta^{(2)} + \boldsymbol{u}^{(0)} \cdot \nabla_P \Theta^{(2)} + w^{(3)} \frac{\partial}{\partial z}\Theta^{(2)} = 0 \, , \tag{3}$$

or in potential vorticity (PV) form

$$\left(\frac{\partial}{\partial t_P} + \boldsymbol{u}^{(0)} \cdot \nabla_P + w^{(3)} \frac{\partial}{\partial z}\right) \frac{f}{\rho^{(0)}} \frac{\partial \Theta^{(2)}}{\partial z} = 0 \, . \tag{4}$$

Here f is the Coriolis parameter, ∇_P the horizontal Nabla operator (acting on the planetary spatial scales), $\rho^{(0)}$ the leading order density and $w^{(3)}$ the first nontrivial term in the vertical velocity expansion. Additionally, we derive a vorticity transport equation on the ten day time scale t_P, which determines the barotropic component of the pressure $\overline{p^{(2)}}^z$. The equation has the form

$$\frac{\partial}{\partial t_P}\left(\frac{1}{a^2 \cos\phi_P} \frac{\partial}{\partial \phi_P} \frac{\cos\phi_P}{f} \frac{\partial}{\partial \phi_P} \overline{p}^z - \frac{\beta}{f^2 a} \frac{\partial}{\partial \phi_P} \overline{p}^z - f \overline{p}^z\right) + F\left(\Theta^{(2)}, \overline{p^{(2)}}^z\right) = 0 \, . \tag{5}$$

The function $F\left(\Theta^{(2)}, \overline{p^{(2)}}^z\right)$ contains terms such as horizontal, vertical advection of vorticity, twisting, solenoidal term and advection of planetary vorticity by the ageostrophic velocities. The component $\overline{p^{(2)}}^z$ can be calculated from the PGEs only if some source terms are added, e.g., friction and surface wind stress. Such approach is applicable to the ocean but not to the atmosphere and the vorticity transport equation (5) gives a possibility to use PG type equations for atmospheric dynamics on the planetary scale.

When in the asymptotic scaling the synoptic processes are also resolved, we obtain a set of two equations. They may be considered as the anelastic analog of Pedlosky's equations (Pedlosky , 1984) for incompressible large scale motions in the ocean. One equation describes the dynamics on the planetary scale of the background stratification $\partial \Theta^{(2)}/\partial z$ This equation is identical to the PV equation in the PG case and there is no influence from the synoptic scales on $\Theta^{(2)}$. The other equation describes the synoptic-scale dynamics. It is a modified quasi-geostrophic (QG) potential vorticity equation, where the synoptic scale PV consists of planetary vorticity, relative vorticity due to the horizontal variations on the synoptic scale of the deviations from the background pressure, denoted by $\pi^{(3)}$, and a stretching term due to the vertical variations of these deviations. In comparison with the classical QG theory there are two additional terms resulting from interactions with the planetary scales. These terms are representing the advection of synoptic scale PV by the planetary scale velocity field (velocities due to the planetary scale variations of $\pi^{(2)}$) and the advection by the synoptic velocity field (velocities due to the synoptic scale variations of $\pi^{(3)}$) of PV due to the planetary scale gradient of $\Theta^{(2)}$.

3 The Planetary Regime with Zonal Jets

Motivated by the observed equator-to-pole temperature differences, we consider in the second regime systematically larger meridional variations of the background potential temperature $\delta\Theta \sim \mathcal{O}(\varepsilon)$. Through thermal wind balance, these variations imply zonal velocities of the order of the jet streams ($\sim 70\,\mathrm{m/s}$) and denoted by $\boldsymbol{u}^{(-1)}$. Due to the high velocities, the advection occurs on the "fast" synoptic time scale t_S. In this case the asymptotic expansion of the potential temperature and of the horizontal wind takes the form

$$\Theta(\underbrace{\varepsilon^2 t}_{t_S}, \underbrace{\varepsilon^3 t}_{t_P}, \lambda_P, \phi_P, z) = 1 + \varepsilon\Theta^{(1)} + \varepsilon^2\Theta^{(2)} + \mathcal{O}\big(\varepsilon^3\big) \ ,$$
$$\boldsymbol{u}(t_S, t_P, \lambda_P, \phi_P, z) = \varepsilon^{-1}\boldsymbol{u}^{(-1)} + \boldsymbol{u}^{(0)} + \varepsilon\boldsymbol{u}^{(1)} + \varepsilon^2\boldsymbol{u}^{(2)} + \mathcal{O}\big(\varepsilon^3\big) \ .$$

To make the discussion as simple as possible we present the results for a Boussinesq fluid and average over the synoptic time t_S. We obtain a PV type equation for $\pi^{(2)}$

$$\begin{aligned}
&\left(\frac{\partial}{\partial t_P} + \boldsymbol{u}^{(0)}\cdot\nabla_P + w^{(3)}\frac{\partial}{\partial z}\right) f\frac{\partial}{\partial z}\left(\frac{\partial\pi^{(2)}/\partial z}{\partial\Theta^{(1)}/\partial z}\right) + \boldsymbol{u}^{(-1)}\cdot\nabla_P\zeta^{(0)} \\
&+\boldsymbol{u}^{(0)}\cdot\nabla_P\zeta^{(-1)} + \zeta^{(-1)}\nabla_P\cdot\boldsymbol{u}^{(0)} + w^{(3)}\frac{\partial}{\partial z}\zeta^{(-1)} + \boldsymbol{e}_z\cdot(\nabla_P w^{(3)}\times\frac{\partial}{\partial z}\boldsymbol{u}^{(-1)}) \\
&-\frac{\tan\phi_P}{a\partial\Theta^{(1)}/\partial z}\frac{\partial\boldsymbol{u}^{(-1)2}}{\partial z}\cdot\nabla_P\frac{\partial\pi^{(2)}}{\partial z} + \beta v_a^{(1)} + \frac{\partial}{\partial z}\left(\frac{f\partial\Theta^{(1)}/\partial\phi_P}{a\partial\Theta^{(1)}/\partial z}v_a^{(1)}\right) = 0\,,
\end{aligned} \tag{6}$$

where

$$\boldsymbol{u}^{(0)} = \frac{1}{f}\boldsymbol{e}_z\times\left(\nabla_P\pi^{(2)} + \frac{u^{(-1)2}\tan\phi_P}{a}\boldsymbol{e}_\phi\right),\ \frac{\partial}{\partial z}\pi^{(2)} = \Theta^{(2)}\,,$$
$$\zeta^{(-1)} = \frac{1}{f}\Delta_P\pi^{(1)} + \frac{u^{(-1)}\cot\phi_P}{a}\,,\zeta^{(0)} = \frac{1}{f}\Delta_P\pi^{(2)} + \frac{u^{(0)}\cot\phi_P}{a} + \frac{1}{a^2\cos\phi}\frac{\partial}{\partial\phi_P}u^{(-1)2}\,,$$
$$\nabla_P\cdot\boldsymbol{u}^{(0)} + \frac{\partial}{\partial z}w^{(3)} = 0\,, v^{(1)} = \underbrace{\frac{1}{af\cos\phi_P}\frac{\partial}{\partial\lambda_P}\pi^{(3)}}_{v_g^{(1)}} + v_a^{(1)}\left(\pi^{(2)}, u^{(-1)}\right)\ .$$

It is interesting to note that the velocity field $\boldsymbol{u}^{(0)}$ (due to $\pi^{(2)}$ variations) is not in geostrophic balance, since metric terms appear in the momentum equation. The PV equation for $\pi^{(2)}$ has additional terms compared with the classical PG equation: horizontal and vertical advection of relative vorticity, divergences term, twisting term and advection by the ageostrophic component $v_a^{(1)}$ of planetary vorticity and of stretching vorticity due to vertical variations of $\Theta^{(1)}$. When the averaged effects from the synoptic scales on the planetary scales are taken into account, spatial averages over the synoptic eddy fluxes appear as a source term in the PV equation. We can conclude that new planetary-synoptic interactions arise in this regime on the fast (synoptic) time scale, because of advection by the large velocities.

4 Conclusions

We presented reduced model equations, which describe the dynamics on planetary spatial scales, i.e., scales of the order of the earth's radius, and account for multiscale planetary-synoptic interactions. Two regimes, associated with different background stratifications, are studied. The first regime (planetary regime) is governed by the PG PV equation and a transport equation for the barotropic component of the pressure, the second regime (planetary regime with zonal jets) by the PG PV equation but with some additional terms such as advection of relative vorticity by the background flow.

After studying the relevance of the two regimes to the atmosphere, the next step will be to incorporate orography and diabatic source terms in the models. The arising vorticity transport equations, similar to that discussed above, can be regarded as an alternative to the temperature-based diagnostic closure for the pressure used in earth system models of intermediate complexity, e.g., CLIMBER (Petoukhov *et al.*, 2000).

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